27032 - Probability Theory

Información del Plan Docente

Academic Year	2017/18
Faculty / School	100 - Facultad de Ciencias
Degree	453 - Degree in Mathematics
ECTS	6.0
Year	4
Semester	Second semester
Subject Type	Optional
Module	---

1.General information

1.1.Introduction

1.2.Recommendations to take this course

1.3.Context and importance of this course in the degree

1.4.Activities and key dates

2.Learning goals

2.1.Learning goals

2.2.Importance of learning goals

3.Aims of the course and competences

3.1.Aims of the course

3.2.Competences

4.Assessment (1st and 2nd call)

4.1.Assessment tasks (description of tasks, marking system and assessment criteria)

5.Methodology, learning tasks, syllabus and resources

5.1.Methodological overview

The learning process will be mainly based on:

-Lectures on the theoretical topics listed in Section 5.3. The explanations will be illustrated by means of a variety of examples, trying to motivate the student`s participation. Interconnections with other mathematical subjects, coming from analysis and applied mathematics will also be discussed. Whenever appropriate, computer presentations will be used.

-Practical sessions in which different exercises and questions will be solved in detail. The students will have in advance the collection of such exercises, in order to facilitate their homework.

-Seminars in which the students will expose the work done. Discussions will be focused on the different ways to face problems, paying special attention to the writing from a mathematical point of view.

-Individual tutorials in order to monitorize the practical work assigned to the student and to correct the ways of working.

Lecture notes, list of exercises, complementary material, references,...will be available for all of the students in the course.

5.2.Learning tasks

In accordance with Section 5.1, the main learning activities are the following:

-Two hours per week of theoretical lectures.

-Two hours per week of practical sessions and seminars.

-Individual tutorials.

-Other learning activities: personal study and work, works in small groups,...

5.3.Syllabus

PART I. PROBABILITY SPACES AND RANDOM VARIABLES

1. PROBABILITY AND MEASURE

Measure: events, sigma algebras of events and non measurable sets. Probability spaces: basic properties. Construction of probability measures.

2. RANDOM VARIABLES

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Random variables: definition and properties. Probability image and distribution function. Relation between distribution functions and random variables. Expectations: classical inequalities. Product spaces and random vectors. Conditional distributions: probability kernels and disintegration of probabilities. Independence of random variables. Borel-Cantelli lemmas.

PART II. TYPES OF CONVERGENCE AND LAWS OF LARGE NUMBERS

3. CONVERGENCE OF RANDOM VARIABLES

Types of convergence: in law, in probability, in mean of order p and almost surely. Relations and limit theorems.

4. LAWS OF LARGE NUMBERS

Weak laws of large numbers. Convergence of random series and strong laws of large numbers. Applications: normal numbers, Montecarlo methods, probabilistic methods in approximation theory.

PART III. CENTRAL LIMIT THEOREMS

5. CHARACTERISTIC FUNCTIONS

Definition and basic properties. Derivatives and moments. Uniqueness, inversion and continuity theorems. Applications in the case of sums of independent random variables.

6. CENTRAL LIMIT THEOREMS

Historical review. Theorems of Moivre-Laplace, Lévy, Liapunov, and Lindeberg-Feller. Applications: construction of asymptotic confidence intervals. Rate of convergence in the central limit theorem: the Berry-Esseen theorem.

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PART IV. INTRODUCTION TO STOCHASTIC PROCESSES

7. MARKOV CHAINS

Definition and construction of a Markov chain. Examples: random walks, queues, and physical biological and economic models. The Chapman-Kolmogorov equation. Clasification of states: periodicity, persistent and transient states. Stopping times: the strong Markov property. Limit distributions and mean recurrence times. Stationary distributions. Ergodic theorems. Applications in physical, biological, and economic models.

8. INTRODUCTION TO STOCHASTIC PROCESSES IN CONTINUOUS TIME

The Kolmogorov consistency theorem. The Poisson process: construction and properties. Non homogeneous and compound Poisson processes: applications. Continuous time Markov chains: applications. Introduction to Brownian motion.

5.4.Course planning and calendar

The schedule of lectures, practical sessions, and seminars will be previously announced in the web page of the faculty, as well as in the web page of the course.

The schedule of practical works by the students will be determined by the teacher.

5.5.Bibliography and recommended resources

- Billingsley, Patrick. Probability and Measure / Patrick Billingsley . - 3rd ed. New York [etc.] : John Wiley, cop. 1995
- Çinlar, Erhan. Probability and Stochastics / Erhan Cinlar . New York : Springer, 2011
- Grimmett, Geoffrey. Probability and Random Processes / Geoffrey Grimmett and David Stirzaker . - 3rd. ed., repr. with corr. Oxford : Oxford University Press, 2004
- Gut, Allan. Probability: A Graduate Course. Springer. 2005
- Norris, J.R.. Markov Chains. Cambridge University Press. 1997
- Resnick, Sidney. Adventures in Stochastic Processes / Sidney Resnick Boston [etc] : Birkhäuser, cop.1992
- Ross, Sheldon M.. Stochastic Processes / Sheldon M. Ross . - 2nd. ed. New York [etc.] : John Wiley and Sons, cop. 1996
- Vélez Ibarrola, Ricardo. Cálculo de Probabilidades 2 / Ricardo Vélez Ibarrola . - [1ª ed.] Madrid : Ediciones Académicas, 2004